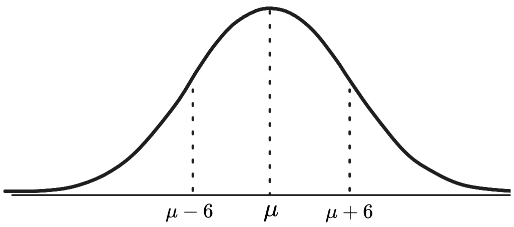
16 - Confidence interval

Population is normally distributed with μ=? and σ=6:

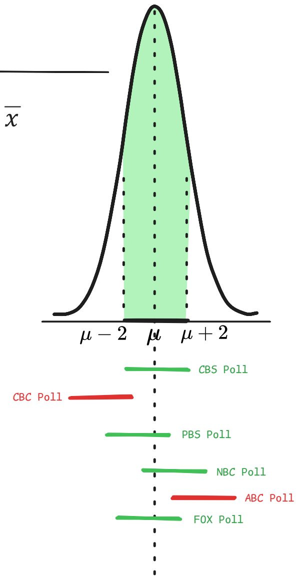


Samples of size n=9 have sample means $\overline{x}$ distributed like $\mu_{\bar{x}} = \mu$ and

$\overline{x}$ lands in interval $(\mu-\sigma, \mu+\sigma)$ for 68% of all resamplings.

Flipped version:
μ lands in interval $(\bar{x}-\sigma, \bar{x}+\sigma)$ for 68% of all resamplings.

"We are 68% confident that μ is in one such given interval."

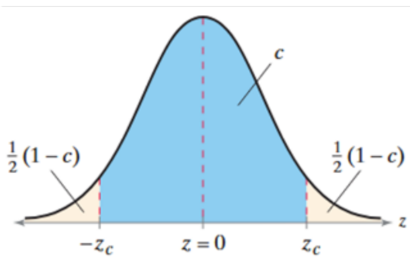


A point estimate is a single number estimate of a population parameter: sample mean $\overline{x}$ estimates population mean μ.

An interval estimate is an interval to estimate a population parameter.

The level of confidence c is the probability that the interval estimate contains the population parameter upon resampling.

Sampling distribution for n≥30 is approx. normal so c is the area under the standard normal curve between critical values $\pm z_c$:



Command in R: $\text{qnorm}\left(\frac{1-c}{2}\right)$

Common critical values:

Level of Confidence	z_c
90%	1.645
95%	1.96
99%	2.575

Facts about Confidence Interval of Mean with σ known

The c-confidence interval for population mean μ obtained from an SRS with sample mean $\overline{x}$ is $\bar{x} - E < \mu < \bar{x} + E$

with margin of error $E = z_c \cdot \frac{\sigma}{\sqrt{n}}$

when the following conditions are met:
- 10% Condition: N≥10n.
- Approx. normal: Population distribution is normal or n≥30.

What we say:
"We are c% confident that μ is in this interval",
"This is the c% confidence interval for μ"

Example 1. Construct a 95% confidence interval for mean age of all Rowan students given pop. is normal, σ=2 years, and an SRS of size n=4 with $\overline{x}$=21 years.

See slides.

Example 2. (Minimal sample size)
Rowan student pop. is normal and σ=2 years. Find min. sample size to be 95% confident the sample mean is within 0.5 year of pop. mean.

See slides.

Facts about Confidence Interval of Proportion

The $\boldsymbol{c}$-confidence interval for pop. proportion $\boldsymbol{p}$ obtained from an SRS with sample proportion $\hat{p}$ is $\hat{p} - E < p < \hat{p} + E$

with margin of error $E = z_c \cdot \sqrt{\dfrac{p(1-p)}{n}}$

when the following conditions are met:
- 10% Condition: N≥10n.
- Large counts: $np \geq 10 \text{ and } n(1-p) \geq 10$

Example 1. Survey of 1550 U.S. adults finds 1054 use FB. Find a 95% confidence interval for the population proportion.

See slides.

To find minimal sample size n to ensure E < some number, solve for n.
If $\hat{p}$ is unknown, use $\hat{p}$ =0.5.

Example 2. (Minimum sample size)
Find min. sample size to be 95% confident the sample proportion is within 2% of true value.

See slides.